

DUAL QCD, EFFECTIVE STRING THEORY, AND REGGE TRAJECTORIES

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We start with an effective field theory containing classical vortex solutions and show that the fluctuations of these vortices are described by an effective string theory. Viewed as a model for long distance QCD, this theory provides a concrete picture of the QCD string as a fluctuating Abrikosov-Nielsen-Olesen vortex of a dual superconductor on the border between type I and type II. We present arguments which suggest that the action of the effective string theory is the Nambu-Goto action, i.e. the rigidity vanishes. We then use this theory to calculate the corrections to classical Regge trajectories due to string fluctuations.

1 Introduction

This talk is dedicated to the memory of Fredrik Zachariasen, a close friend and colleague of one of us (MB) for over forty years. The work we will describe today was inspired by a fifteen year collaboration with Fred and Jim Ball. Fred invented the name “dual QCD,” and the sound of these words evokes a vivid memory of Fred, and of how much the collaboration meant to me.

We first review the results of dual QCD, which is an effective field theory for long distance QCD containing classical vortex solutions. We then show that the fluctuations of these superconducting vortices are described by an effective string theory. We present arguments which suggest that the action of this effective string theory is the Nambu-Goto action. Finally, we apply the results of a semi-classical expansion of the effective theory to calculate corrections to classical Regge trajectories due to string fluctuations.

2 The Transformation from Fields to Strings

In the dual superconductor picture of confinement^{1,2}, a dual Meissner effect confines electric color flux (\mathbf{Z}_3 flux) to narrow tubes connecting quark-antiquark pairs. Calculations with a concrete version³ of this model (dual QCD) has been compared both with experimental data and with Monte Carlo simulations of QCD⁴. To a good approximation, the dual Abelian Higgs model (with a suitable color factor) can be used to describe these calculations. The Lagrangian \mathcal{L}_{eff} describing long distance QCD in the dual superconductor pic-

ture then has the form:

$$\mathcal{L}_{\text{eff}} = \frac{4}{3} \left\{ -\frac{1}{4} G_{\mu\nu}^2 - \frac{1}{2} |(\partial_\mu - igC_\mu)\phi|^2 - \frac{\lambda}{4} (|\phi|^2 - \phi_0^2)^2 \right\}. \quad (1)$$

The potentials C_μ are dual potentials, and ϕ is a complex Higgs field carrying monopole charge, whose vacuum expectation value ϕ_0 is nonvanishing. All particles are massive: $M_\phi = \sqrt{2\lambda}\phi_0$, $M_C = g\phi_0$. The dual coupling constant is $g = \frac{2\pi}{e}$, where e is the Yang–Mills coupling constant. The potentials C_μ couple to the $q\bar{q}$ pair via $G_{\mu\nu}^S$, a Dirac string whose ends are a source and a sink of electric color flux,

$$G_{\mu\nu}^S = -e \int d^2\xi \frac{1}{2} \epsilon^{ab} \epsilon_{\mu\nu\alpha\beta} \frac{\partial \tilde{x}^\alpha}{\partial \xi^a} \frac{\partial \tilde{x}^\beta}{\partial \xi^b} \delta^{(4)}(x^\mu - \tilde{x}^\mu(\xi)). \quad (2)$$

The field strength $G_{\mu\nu}$ is expressed in terms of C_μ and $G_{\mu\nu}^S$,

$$G_{\mu\nu} = \partial_\mu C_\nu - \partial_\nu C_\mu + G_{\mu\nu}^S. \quad (3)$$

The effect of the string is to create a flux tube (Abrikosov–Nielsen–Olesen (ANO) vortex⁵) along some line L connecting the quark–antiquark pair, on which the dual Higgs field ϕ must vanish. As the pair moves, the line L sweeps out a space time surface \tilde{x}^μ , whose boundary is the loop Γ formed by the world lines of the quark and antiquark trajectories. (See Fig. 1) The monopole field ϕ vanishes on the surface $\tilde{x}^\mu(\xi)$ parameterized by ξ^a , $a = 1, 2$:

$$\phi(\tilde{x}^\mu(\xi)) = 0. \quad (4)$$

Eq. (4) determines the location \tilde{x}^μ of the ANO vortex of the field configuration $\phi(x^\mu)$. The long distance $q\bar{q}$ interaction is determined by the Wilson loop $W[\Gamma]$,

$$W[\Gamma] = \int \mathcal{D}C_\mu \mathcal{D}\phi \mathcal{D}\phi^* e^{iS[C_\mu, \phi]}. \quad (5)$$

The functional integration goes over all field configurations containing a vortex sheet whose boundary is Γ , and the action is

$$S[C_\mu, \phi] = \int d^4x \mathcal{L}_{\text{eff}}. \quad (6)$$

Previous calculations of $W[\Gamma]$ were carried out in the classical approximation (corresponding to a flat vortex sheet $\tilde{x}^\mu(\sigma)$), and showed³ that the

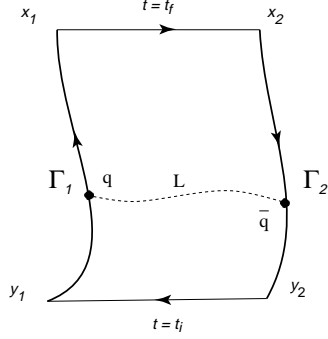


Figure 1: The Loop Γ

Landau–Ginzburg parameter λ/g^2 is approximately equal to $\frac{1}{2}$. Both the vector and scalar particles have the same mass $M = g\phi_0 \approx 910$ MeV, and the flux tube radius is $a = \frac{\sqrt{2}}{M}$.

The classical approximation neglects the effect of fluctuations in the shape of the flux tube on the $q\bar{q}$ interaction. To take into account these fluctuations, we must evaluate the functional integral (5) beyond the classical approximation. We carry out this integration in two steps: (1) We fix the location of a vortex sheet \tilde{x}^μ , and integrate only over field configurations for which $\phi(x^\mu)$ vanishes on \tilde{x}^μ . (2) We integrate over all possible vortex sheets. To implement this procedure, we introduce into the functional integral (5) the factor one, written in the form

$$1 = J[\phi] \int \mathcal{D}\tilde{x}^\mu \delta[\text{Re}\phi(\tilde{x}^\mu(\xi))] \delta[\text{Im}\phi(\tilde{x}^\mu(\xi))] . \quad (7)$$

The integration $\mathcal{D}\tilde{x}^\mu$ is over the four functions $\tilde{x}^\mu(\xi)$. The functions $\tilde{x}^\mu(\xi)$ are a particular parameterization of the worldsheet \tilde{x}^μ .

The expression (7) implies that the string worldsheet \tilde{x}^μ , determined by the δ functions, is the surface of the zeros of the field ϕ . Inserting (7) into (5) puts the Wilson loop in the form

$$W[\Gamma] = \int \mathcal{D}\phi^* \mathcal{D}\phi \mathcal{D}C^\mu e^{iS[\phi, C]} J[\phi] \int \mathcal{D}\tilde{x}^\mu \delta[\text{Re}\phi(\tilde{x}^\mu(\xi))] \delta[\text{Im}\phi(\tilde{x}^\mu(\xi))] . \quad (8)$$

We then reverse the order of the field integration and the string integration over surfaces $\tilde{x}^\mu(\xi)$,

$$W[\Gamma] = \int \mathcal{D}\tilde{x}^\mu \int \mathcal{D}\phi^* \mathcal{D}\phi \mathcal{D}C^\mu J[\phi] \delta[\text{Re}\phi(\tilde{x}^\mu(\xi))] \delta[\text{Im}\phi(\tilde{x}^\mu(\xi))] e^{iS[\phi, C]} . \quad (9)$$

In Eq. (8), the δ functions fix \tilde{x}^μ to lie on the surface of the zeros of a given field ϕ , while in Eq. (9), they restrict the field ϕ to vanish on a given surface \tilde{x}^μ . The integral over ϕ in Eq. (9) is therefore restricted to functions ϕ which vanish on \tilde{x}^μ , in contrast to the integral over ϕ in Eq. (8), in which ϕ can be any function.

3 Factorization of the Jacobian

The Jacobian $J[\phi]$ in Eq. (9) is evaluated for field configurations ϕ which vanish on a particular surface \tilde{x}^μ . We make this explicit by writing (7) as

$$J[\phi, \tilde{x}^\mu]^{-1} = \int \mathcal{D}\tilde{y}^\mu \delta[\text{Re}\phi(\tilde{y}^\mu(\tau))] \delta[\text{Im}\phi(\tilde{y}^\mu(\tau))] , \quad (10)$$

where \tilde{y}^μ is some other string worldsheet, distinct from \tilde{x}^μ .

The δ functions in (10) select surfaces $\tilde{y}^\mu(\tau)$ which lie in a neighborhood of the surface $\tilde{x}^\mu(\xi)$ of the zeros of ϕ . We separate $\tilde{y}^\mu(\tau)$ into components lying on the surface $\tilde{x}^\mu(\xi)$ and components lying along vectors $n_\mu^A(\xi)$ normal to $\tilde{x}^\mu(\xi)$ at the point ξ :

$$\tilde{y}^\mu(\tau) = \tilde{x}^\mu(\xi(\tau)) + y_\perp^A(\xi(\tau)) n_\mu^A(\xi(\tau)) . \quad (11)$$

The point $\tilde{x}^\mu(\xi(\tau))$ is the point on the surface $\tilde{x}^\mu(\xi)$ lying closest to $\tilde{y}^\mu(\tau)$, and

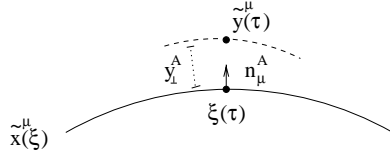


Figure 2: Worldsheets and normal vectors

the magnitude of $y_\perp^A(\xi(\tau))$ is the distance from $\tilde{y}^\mu(\tau)$ to $\tilde{x}^\mu(\xi(\tau))$ (see Fig. 2).

We now exhibit the factorization of the Jacobian. Making the change of coordinates $\tilde{y}^\mu(\tau) \rightarrow (\xi(\tau), y_\perp^A(\xi))$ gives

$$J[\phi, \tilde{x}^\mu]^{-1} = \int \mathcal{D}\xi \mathcal{D}y_\perp^A \text{Det}_\tau[\sqrt{-g}] \delta[\text{Re}\phi(\tilde{x}^\mu + y_\perp^A n_\mu^A)] \delta[\text{Im}\phi(\tilde{x}^\mu + y_\perp^A n_\mu^A)] , \quad (12)$$

where $\sqrt{-g}$ is the square root of the determinant of the induced metric

$$g_{ab} = \frac{\partial \tilde{x}^\mu}{\partial \xi^a} \frac{\partial \tilde{x}^\mu}{\partial \xi^b} . \quad (13)$$

Eq. (12) has the form:

$$J[\phi, \tilde{x}]^{-1} = \int \mathcal{D}\xi(\tau) \text{Det}_\tau [\sqrt{-g}] J_\perp[\phi, \tilde{x}^\mu(\xi(\tau))]^{-1}, \quad (14)$$

where

$$J_\perp[\phi, \tilde{x}^\mu]^{-1} = \int \mathcal{D}y_\perp^A \delta [\text{Re}\phi (\tilde{x}^\mu + y_\perp^A n_A^\mu)] \delta [\text{Im}\phi (\tilde{x}^\mu + y_\perp^A n_A^\mu)] \quad (15)$$

contains all the dependence on ϕ . The Jacobian J_\perp is the Faddeev-Popov determinant for the degrees of freedom y_\perp^A which move the string.

Since J_\perp is independent of the parameterization $\xi(\tau)$, the Jacobian factors into two parts:

$$J[\phi, \tilde{x}]^{-1} = J_\parallel[\tilde{x}]^{-1} J_\perp[\phi, \tilde{x}]^{-1}, \quad (16)$$

where

$$J_\parallel[\tilde{x}]^{-1} = \int \mathcal{D}\xi \text{Det}_\tau [\sqrt{-g}]. \quad (17)$$

The string part J_\parallel of the Jacobian arises from the parameterization degrees of freedom. In the following section, we will use J_\parallel to fix the reparameterization degrees of freedom.

4 Effective String Theory of Vortices

Inserting the factorized form (16) of $J[\phi]$ into the expression (9) for $W[\Gamma]$ gives the Wilson Loop the form

$$W[\Gamma] = \int \mathcal{D}\tilde{x}^\mu J_\parallel[\tilde{x}] e^{iS_{\text{eff}}}, \quad (18)$$

where the action S_{eff} of the effective string theory is given by

$$e^{iS_{\text{eff}}[\tilde{x}^\mu(\xi)]} = \int \mathcal{D}\phi^* \mathcal{D}\phi \mathcal{D}C^\mu J_\perp[\phi] \delta [\text{Re}\phi(\tilde{x}^\mu(\xi))] \delta [\text{Im}\phi(\tilde{x}^\mu(\xi))] e^{iS}. \quad (19)$$

The string action (19) was obtained previously by Gervais and Sakita⁶. The novel feature of our result is the string integration measure of the Wilson loop (18).

Any surface \tilde{x}^μ has only two physical degrees of freedom. The other two degrees of freedom represent the invariance of the surface under coordinate reparameterizations. We fix the coordinate reparameterization symmetry by

choosing a particular “representation” x^μ of the surface, which depends on two functions $f^1(\xi)$, $f^2(\xi)$,

$$x^\mu(\xi) = x^\mu[f^1(\xi), f^2(\xi), \xi]. \quad (20)$$

Any physical surface can be expressed in terms of x^μ by a suitable choice of f^1 and f^2 . In particular, the worldsheet $\tilde{x}^\mu(\xi)$ appearing in the integral (18) can be written in terms of a reparameterization $\tilde{\xi}(\xi)$ of the representation x^μ ,

$$\tilde{x}^\mu(\xi) = x^\mu[f^1(\tilde{\xi}(\xi)), f^2(\tilde{\xi}(\xi)), \tilde{\xi}(\xi)]. \quad (21)$$

The four degrees of freedom in $\tilde{x}^\mu(\xi)$ are replaced by two physical degrees of freedom $f^1(\xi)$, $f^2(\xi)$ and two reparameterization degrees of freedom $\tilde{\xi}(\xi)$.

We can write the integral over $\tilde{x}^\mu(\xi)$ in (18) in terms of integrals over $f^1(\xi)$, $f^2(\xi)$ and $\tilde{\xi}(\xi)$,

$$\mathcal{D}\tilde{x}^\mu = \text{Det} \left[\tilde{t}_{\mu\nu} \sqrt{-g} \frac{\partial x^\mu}{\partial f^1} \frac{\partial x^\nu}{\partial f^2} \right] \mathcal{D}f^1 \mathcal{D}f^2 \mathcal{D}\tilde{\xi}, \quad (22)$$

where

$$\tilde{t}_{\mu\nu} = \frac{1}{2} \epsilon_{\mu\nu\alpha\beta} \frac{\epsilon^{ab}}{\sqrt{-g}} \frac{\partial x^\alpha}{\partial \xi^a} \frac{\partial x^\beta}{\partial \xi^b}, \quad (23)$$

is the antisymmetric tensor normal to the worldsheet.

With the parameterization (21) of \tilde{x}^μ , the path integral (18) takes the form

$$W[\Gamma] = \int \mathcal{D}\tilde{\xi} \mathcal{D}f^1 \mathcal{D}f^2 \text{Det} \left[\tilde{t}^{\mu\nu} \frac{\partial x^\mu}{\partial f^1} \frac{\partial x^\nu}{\partial f^2} \right] \text{Det}[\sqrt{-g}] J_{\parallel} e^{iS_{\text{eff}}}. \quad (24)$$

Due to the invariance of the theory under coordinate reparameterizations, the only term in (24) which depends on $\tilde{\xi}$ is the determinant of $\sqrt{-g}$. When we bring the terms which are independent of $\tilde{\xi}$ outside of the integral, the path integral becomes

$$W[\Gamma] = \int \mathcal{D}f^1 \mathcal{D}f^2 \text{Det} \left[\tilde{t}^{\mu\nu} \frac{\partial x^\mu}{\partial f^1} \frac{\partial x^\nu}{\partial f^2} \right] J_{\parallel} e^{iS_{\text{eff}}} \int \mathcal{D}\tilde{\xi} \text{Det}[\sqrt{-g}]. \quad (25)$$

The remaining integral over reparameterizations $\tilde{\xi}$ is equal to J_{\parallel}^{-1} , defined by (17), and is canceled by the explicit factor of J_{\parallel} appearing in (25). This means we do not need to evaluate J_{\parallel} , and can avoid the complications inherent in evaluating the integral over reparameterizations of the string coordinates. The anomalies⁷ produced in string theory by evaluating this integral are not

present, and there is no Polchinski–Strominger term^{8,9} in the theory. Eq. (25) gives the final result for the Wilson loop

$$W[\Gamma] = \int \mathcal{D}f^1 \mathcal{D}f^2 \text{Det} \left[\tilde{t}_{\mu\nu} \frac{\partial x^\mu}{\partial f^1} \frac{\partial x^\nu}{\partial f^2} \right] e^{iS_{\text{eff}}}, \quad (26)$$

as an integration over two function $f^1(\xi)$ and $f^2(\xi)$, the physical degrees of freedom of the string. “Gauge fixing” the reparameterization symmetry has produced a Faddeev–Popov determinant.

5 The Effective Action

The action (19) of the effective string theory gives the action $S_{\text{eff}}(\tilde{x}^\mu)$ of the effective string theory (26) as an integral over field configurations which have a vortex fixed at \tilde{x}^μ . Since the vortex theory (5) is an effective long distance theory, the path integral (5) for $W[\Gamma]$, written in terms of the fields of the Abelian Higgs model, is cut off at a scale Λ which is on the order of the mass M of the dual gluon. Furthermore, the integration (18) over \tilde{x}^μ includes all the long distance fluctuations of the theory. Therefore, the path integral (6) contains neither short distance nor long distance fluctuations, and is determined by minimizing the field action $S[\tilde{x}^\mu, \phi, C_\mu]$ for a fixed position of the vortex sheet:

$$S_{\text{eff}}[\tilde{x}^\mu] = S[\tilde{x}^\mu] \equiv S[\tilde{x}^\mu, \phi^{\text{class}}, C_\mu^{\text{class}}], \quad \phi^{\text{class}}(\tilde{x}^\mu) = 0. \quad (27)$$

The fields ϕ^{class} and C_μ^{class} are the solutions of the classical equations of motion, subject to the boundary condition $\phi(\tilde{x}^\mu) = 0$.

To evaluate $W[\Gamma]$, we need to know the classical action $S[\tilde{x}^\mu]$ for strings of length R and radius of curvature R_V greater than the flux tube radius a (see Fig. 3). In the case of long straight strings, $a/R \ll 1$, $a/R_V \ll 1$, the classical

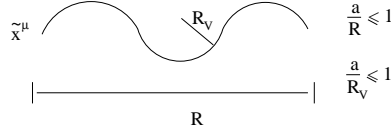


Figure 3: Length scales of the string

action S becomes the Nambu–Goto action S_{NG} ,

$$S[\tilde{x}^\mu] = S_{\text{NG}} \equiv -\sigma \int d^2\xi \sqrt{-g}. \quad (28)$$

In the next two sections, we consider separately long bent strings ($R \rightarrow \infty$) and short straight strings ($R_V \rightarrow \infty$). We present arguments that suggest that the action (28) is a good approximation to $S_{\text{eff}}[\tilde{x}^\mu]$ in both these situations.

6 Long Bent Strings; the Rigidity

For long bent strings ($R \rightarrow \infty$), the leading correction to the Nambu–Goto action is the curvature term

$$S_{\text{curvature}} = -\beta \int d^2\xi \sqrt{-g} \left(-\nabla^2 \tilde{x}^\mu \right)^2 \sim \frac{a^2}{R_V^2} S_{NG}, \quad (29)$$

where

$$-\nabla^2 \tilde{x}^\mu = -\frac{1}{\sqrt{-g}} \frac{\partial}{\partial \xi^a} \sqrt{-g} g^{ab} \frac{\partial}{\partial \xi^b} \tilde{x}^\mu. \quad (30)$$

The calculation of the “rigidity” β determining the size of $S_{\text{curvature}}$ has been considered by a number of authors¹⁰, but its value for a superconductor on the I–II border was never calculated. We conjecture that $\beta = 0$ on the basis of analytic results for an infinite Nielsen–Olesen flux tube on the (I/II) border obtained by de Vega and Schaposnik¹¹. We now briefly describe their results.

We denote the color field by \vec{D} :

$$\vec{D} = D_{\text{FT}}(r) \hat{\mathbf{e}}_z, \quad D_{\text{FT}}(r) = G_{\theta r}(r), \quad (31)$$

where r and θ are the radial coordinates in a plane perpendicular to the axis of the flux tube, which lies along the z axis. The field \vec{D} in a tube segment of length R is indicated in Figure 4. We separate the action S into a Higgs contribution S_ϕ and a gauge contributions S_g ,

$$S = S_\phi + S_g, \quad (32)$$

where

$$S_\phi = \int d^4x \left[-\frac{1}{2} (\partial_\mu |\phi|)^2 - \frac{\lambda}{4} (|\phi|^2 - \phi_0^2)^2 \right], \quad (33)$$

and

$$S_g = \int d^4x \left[-\frac{1}{2} G_{\mu\nu}^2 - \frac{1}{2} g^2 C_\mu^2 |\phi|^2 \right]. \quad (34)$$

For a straight flux tube,

$$S_\phi = -\sigma_\phi \int d^2\xi \sqrt{-g}, \quad S_g = -\sigma_g \int d^2\xi \sqrt{-g}, \quad (35)$$

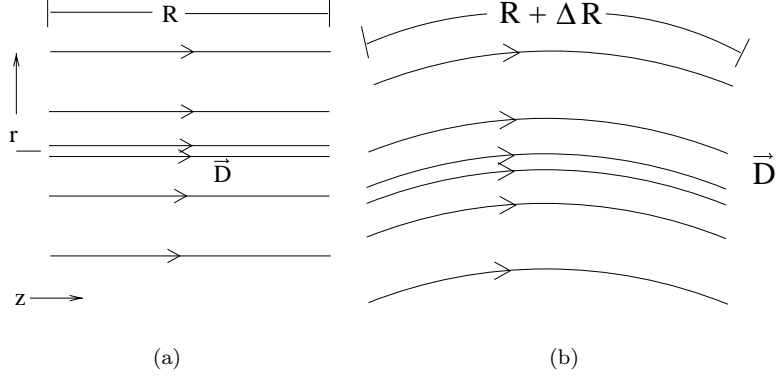


Figure 4: Field lines for (a) a straight and (b) a bent vortex

and the string tension σ is the sum $\sigma = \sigma_\phi + \sigma_g$. Furthermore, the difference

$$S_g - S_\phi = \int d^4x T_{\theta\theta}, \quad (36)$$

where $T_{\theta\theta}$ is the $\theta\theta$ component of the stress tensor.

de Vega and Schaposnik showed that for a superconductor on the (I/II) border

$$\sigma_\phi = \sigma_g = \sigma/2 = \frac{\pi}{2}\phi_0^2, \text{ so that } S_\phi - S_g = 0. \quad (37)$$

They also showed the components of the stress tensor in the plane perpendicular to the flux tube vanish, $T_{rr} = T_{\theta\theta} = 0$. The repulsion between the lines of force of the gauge field is compensated by the attraction produced by the Higgs fields. In other words, there are no “bonds” perpendicular to the axis of a straight flux tube.

Now suppose the flux tube is bent slightly, so that $R \rightarrow R + \Delta R$ as indicated in Figure 4. Since no perpendicular bonds are stretched or compressed, the corresponding change in the energy should be the string tension multiplied by the change in length. That is, the curvature term, which in a sense represents the attraction or repulsion between neighboring parts of the string, should vanish.

We now present some formal arguments which support this conjecture. We consider a general vortex sheet \tilde{x}^μ (see Fig. 3), and again separate the action into a Higgs contribution (33) and a gauge contribution (34). Using the

classical equations of motion for the gauge field C^μ , we can write the action S_g in the alternate form

$$S_g = -\frac{1}{4} \int d^4x G_{\mu\nu}^S G^{\mu\nu}. \quad (38)$$

Since $G_{\mu\nu}^S$ is nonzero only on the worldsheet \tilde{x}^μ , Eq. (38) can be written as an integral over the worldsheet $\tilde{x}^\mu(\xi)$. The field tensor $G_{\mu\nu}$ is a function of the spacetime point x^μ and a functional of the worldsheet \tilde{x}^μ . We define the color field $D(\xi, \tilde{x}^\mu)$ at the point $x^\mu = \tilde{x}^\mu(\xi)$ on the worldsheet by the equation

$$\frac{1}{2} D^2(\xi, \tilde{x}^\mu) = \frac{1}{4} G_{\mu\nu}^2(x^\mu, \tilde{x}^\mu) \Big|_{x^\mu = \tilde{x}^\mu(\xi)}. \quad (39)$$

Eq. (38) for S_g can then be written in the form

$$S_g = -\frac{e}{2} \int d^2\xi \sqrt{-g} D(\xi, \tilde{x}^\mu). \quad (40)$$

We now find the change in the action when we vary the position of the worldsheet \tilde{x}^μ . Since the classical solution is a stationary point of the action, only the explicit dependence of S on \tilde{x}^μ contributes to δS . This explicit dependence is only present in the term $\frac{1}{4} \int d^4x G_{\mu\nu}^2$ which contains $G_{\mu\nu}^S$. Hence, when $\tilde{x}^\mu \rightarrow \tilde{x}^\mu + \delta\tilde{x}^\mu$, then $S \rightarrow S + \delta S$, where

$$\delta S = S[\tilde{x}^\mu + \delta\tilde{x}^\mu] - S[\tilde{x}^\mu] = -\frac{1}{2} \int d^4x \delta G_{\mu\nu}^S G^{\mu\nu}. \quad (41)$$

Alternately, we can use (32) and (38) to evaluate δS explicitly,

$$\delta S = \delta S_\phi - \frac{1}{4} \int d^4x \delta G_{\mu\nu}^S G^{\mu\nu} - \frac{1}{4} \int d^4x G_{\mu\nu}^S \delta G^{\mu\nu}, \quad (42)$$

or

$$-\frac{1}{4} \int d^4x G_{\mu\nu}^S \delta G^{\mu\nu} = \frac{1}{2} \delta S - \delta S_\phi = \frac{1}{2} \delta (S_g - S_\phi). \quad (43)$$

Writing (43) in terms of δD gives

$$e \int d^2\xi \sqrt{-g} \frac{\delta D(\xi, \tilde{x}^\mu)}{\delta \tilde{x}^\mu} = -\frac{\delta (S_g - S_\phi)}{\delta \tilde{x}^\mu}. \quad (44)$$

Eq. (44) relates the change in the color field D on the string to the change of the difference $S_g - S_\phi$ as the position \tilde{x}^μ of the worldsheet is varied.

In the limit of a flat sheet, $D(\xi, \tilde{x}^\mu)$ is a constant independent of the position ξ on the worldsheet,

$$D(\xi, \tilde{x}^\mu) = D_{\text{FT}} \equiv D_{\text{FT}}(r = 0), \quad (45)$$

and Eq. (40) for S_g reduces to the Nambu–Goto action with $\sigma_g = \frac{e}{2} D_{\text{FT}}$. We can estimate the size of the corrections to (45) for slightly bent sheets, by using (35) to evaluate the right hand side of (44). This gives

$$\begin{aligned} e \int d^2 \xi \sqrt{-g} \frac{\delta D(\xi, \tilde{x}^\mu)}{\delta \tilde{x}^\mu} &\approx (\sigma_g - \sigma_\phi) \frac{\delta}{\delta \tilde{x}^\mu} \int d^2 \xi \sqrt{-g} \\ &= (\sigma_g - \sigma_\phi) \int d^2 \xi \sqrt{-g} (-\nabla^2 \tilde{x}^\mu). \end{aligned} \quad (46)$$

The quantity $-\nabla^2 \tilde{x}^\mu$ is proportional to the extrinsic curvature, which is of order a/R_V , so that the correction to the flat sheet limit (45) is of order a/R_V . However, for a superconductor on the (I–II) border, $\sigma_g - \sigma_\phi = 0$, so that the flat sheet expressions (35) and (45) for S_ϕ , S_g , and $D(\xi, \tilde{x}^\mu)$ are compatible with the relations (40) and (44) for sheets with nonvanishing extrinsic curvature. This suggests that the rigidity vanishes. Eq. (46) is the formal manifestation of the heuristic argument given earlier that bending a flux tube on the (I/II) border does not produce a perturbation of the order a/R_V .

7 Short Straight Strings

Recent numerical studies¹² of the classical equations for a flat sheet have shown that for a superconductor on the border the Nambu–Goto action (28) remains a good approximation for short straight strings ($a/R \leq 1$, $R_V = \infty$). We now show that this result is compatible with equations (40) and (44), written in a form appropriate to a straight string of finite length R . In this situation, the action $S = TV(R)$, where T is the elapsed time and $V(R)$ is the static potential between a $q\bar{q}$ pair separated by a distance R . We divide $V(R)$ into a Coulomb part V^{Coulomb} and a nonperturbative part V^{NP} :

$$V(R) = V^{\text{Coulomb}}(R) + V^{\text{NP}}(R). \quad (47)$$

We then separate the gauge and Higgs contributions to $V^{\text{NP}}(R)$,

$$V^{\text{NP}}(R) = V_\phi + V_g^{\text{NP}}. \quad (48)$$

Then Eq. (40) becomes:

$$V_g^{\text{NP}} = \frac{e}{2} \int_{-R/2}^{R/2} dz D^{\text{NP}}(z, R), \quad (49)$$

where $D^{\text{NP}}(z, R)$ is the nonperturbative part of the color field at a point z on the string. The change $\delta\tilde{x}^\mu$ in the string is replaced by the change δR in its length, so Eq. (44) becomes

$$e \int_{-R/2}^{R/2} dz \frac{\partial D^{\text{NP}}(z, R)}{\partial R} = \frac{d}{dR} [V_g^{\text{NP}}(R) - V_\phi(R)] . \quad (50)$$

At large R , the nonperturbative fields become the fields of an infinitely long flux tube, so that

$$D^{\text{NP}}(z, R) \rightarrow D_{\text{FT}}, \text{ for } -\frac{R}{2} \leq z \leq \frac{R}{2}, \quad V_g^{\text{NP}} \rightarrow \sigma_g R, \quad V_\phi \rightarrow \sigma_\phi R. \quad (51)$$

Using the large R potentials on the right hand side of (50) gives

$$\frac{e}{2} \int_{-R/2}^{R/2} dz \frac{\partial D^{\text{NP}}(z, R)}{\partial R} \approx (\sigma_g - \sigma_\phi) . \quad (52)$$

Eq. (52) shows that the color field $D^{\text{NP}}(z, R)$ at a given point z on the string depends upon the length R of the string. Therefore, the field on the interior of the string can, in general, no longer remain a constant, D_{FT} , as R becomes smaller. However, for a superconductor on the (I/II) border the right hand side of Eq. (52) vanishes, so that the solution $D^{\text{NP}}(z, R) = D_{\text{FT}}$ is also consistent at smaller R . In fact, the potential V_g^{NP} , which is the product of the field on the string and the length R of the string, can remain proportional to R for smaller values of R . The same is true for $V_\phi(R)$, and the potential $V^{\text{NP}} = \sigma R$ is consistent with the constraint (50) for all values of R .

8 Regge Trajectories for Light Mesons

In this section we calculate corrections to classical Regge trajectories due to string fluctuations. We take the action of the effective string theory to be the Nambu–Goto action for all R and R_V such that $a/R \leq 1$ and $a/R_V \leq 1$,

$$S_{\text{eff}} = -\sigma \int d^2\xi \sqrt{-g}. \quad (53)$$

Combining (26) and (53) gives the Wilson loop $W[\Gamma]$ of the effective string theory,

$$W[\Gamma] = \int \mathcal{D}f^1 \mathcal{D}f^2 \text{Det} \left[\tilde{t}_{\mu\nu} \frac{\partial x^\mu}{\partial f^1} \frac{\partial x^\nu}{\partial f^2} \right] e^{-i\sigma \int d^2\xi \sqrt{-g}}. \quad (54)$$

Let Γ be the loop generated by the worldlines of an equal mass quark–antiquark pair separated by a distance R , and rotating with angular velocity ω . The

velocity of the quarks is $v = \omega R/2$. The effective Lagrangian L for the quark-antiquark pair is given by

$$L(R, \omega) = -2m\sqrt{1-v^2} + L^{\text{string}}(R, \omega), \quad (55)$$

where

$$L^{\text{string}} = \frac{-i}{T} \log W[\Gamma]. \quad (56)$$

Since both R and ω are fixed, L^{string} is time independent.

We evaluate L^{string} by carrying out a semiclassical expansion of $W[\Gamma]$ about the classical rigid rotating string solution \bar{x}^μ . This expansion gives

$$L^{\text{string}} = L_{\text{cl}}^{\text{string}} + L_{\text{fluc}}, \quad (57)$$

where

$$L_{\text{cl}}^{\text{string}} = -\frac{\sigma}{T} \int d^2\xi \sqrt{-g}, \quad (58)$$

and L_{fluc} is the contribution of the long wavelength transverse vibrations of the rotating string, which is analogous to the result¹³ of Lüscher for static strings.

We can write (55) and (57) in the form

$$L(R, \omega) = L_{\text{class}} + L_{\text{fluc}}, \quad (59)$$

with

$$L_{\text{class}} = -2m\sqrt{1-v^2} - \sigma \int_{-R/2}^{R/2} dr \sqrt{1-r^2\omega^2}. \quad (60)$$

The effective Lagrangian (59) determines the angular momentum J and the energy E of the rotating quarks,

$$J = \frac{\partial L}{\partial \omega}, \quad E = \omega \frac{\partial L}{\partial \omega} - L. \quad (61)$$

The “equation of motion” $\frac{\partial L}{\partial R} = 0$ determines the frequency of rotation ω as a function of R , $\omega = \omega(R)$. We have calculated L_{fluc} , and find that $L_{\text{fluc}} \ll L_{\text{cl}}^{\text{string}}$ for large R . The semiclassical expansion is then justified, and L_{fluc} can be treated as a perturbation. The energy $E(J)$ is then related to the energy $E_{\text{class}}(J)$, calculated in the absence of string fluctuations, by the equation

$$E(J) = E_{\text{class}}(J) - L_{\text{fluc}}. \quad (62)$$

Eq. (62) gives the Regge trajectory

$$J = \frac{E^2}{2\pi\sigma} - \sqrt{\frac{E}{\pi^3 m}} \left[\ln \left(\frac{Mm}{\sigma} \right) + 1 \right] - \frac{4}{3\sigma} \sqrt{\frac{m^3 E}{\pi}} + \frac{7}{12} + O \left(\frac{\ln E}{\sqrt{E}} \right). \quad (63)$$

The short wavelength fluctuations are cut off at a wavelength $\lambda \sim a \sim 1/M$.

In Fig. 5, we plot the Regge trajectory (63) using the values $M = 910$

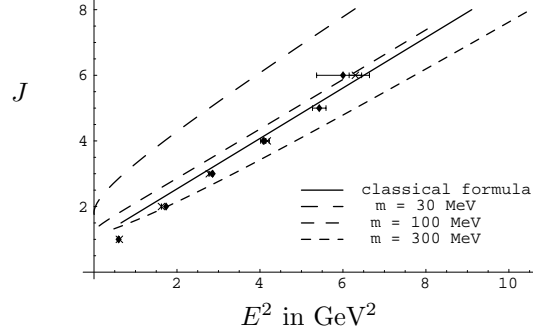


Figure 5: ρ - ω Regge trajectory

MeV and $\sigma = (455 \text{ MeV})^2$ obtained previous fits of heavy quark potentials³. We have chosen quark masses of 30, 100, and 300 MeV. For comparison, we also plot the classical formula $J = E^2/2\pi\sigma$. The plotted points are observed particles on the ω and ρ trajectories. We have added one to the value of J on the plot to account for the spin of the quarks. We have chosen the range of values for the quark masses in Fig. 5 in order to give a qualitative picture of the dependence of the Regge trajectory on the quark mass. Since (63) does not include the contribution of quark fluctuations to the Regge trajectory, this formula is incomplete.

9 Conclusions

1. The dual superconductor description of long distance QCD yields the effective string theory (26) of superconducting vortices.
2. We have presented arguments which suggest that the action of the effective string theory is the Nambu–Goto action (28).
3. The semiclassical expansion of the effective string theory, including the fluctuations of the vortex, gives the result (63) for Regge trajectories. We are in the process of taking into account the quantum fluctuations of the quark degrees of freedom in order to complete the calculation of the semiclassical corrections to Regge trajectories.

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